XI. *On Thermal Radiation in Absolute Measure.**

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[Plates 17, 18.]

In 1884† and in 1887‡ I had the honour of submitting to the Royal Society the results of certain experimental determinations of thermal emission in absolute measure. In these experiments metallic wires heated by an electric current were used. Much remains to be done by the methods and with the apparatus then employed; and I

* [Since this paper was written, and since it was accepted by the Royal Society, a paper has appeared in the 'Philosophical Transactions,' 1893, A., vol. 183, by Professor W. E. Ayrton, F.R.S., and Mr. H. Kilgour, "On Thermal Emissivity of Thin Wires." In their preliminary remarks the authors, referring to my previous papers (among others), say: "But it was not part of these investigations to determine the change in the emissivity that is produced by change in the shape and size of the cooling body."

I cannot avoid pointing out that this remark is wholly erroneous. My work on thermal emission, which was commenced on November 9, 1883, had altogether for its object, in the first instance, the finding of the effect of dimensions and form of the cooling body on the thermal emission; and the question of dimensions has never been lost sight of by me, and has, I think, been referred to in every paper I have published on this subject.

In the 'Electrician' for April 19, 1884, there appeared a short account of my experiments, which were commenced as an experimental counterpart to a paper on "The Efficiency of Clothing," read by Lord Kelvin to the Royal Society of Edinburgh, on March 3, 1883. I found, however, that Professor G. Forbes had already (British Association, 1882, and 'Electrician,' September, 1882) given important conclusions, derived theoretically, as to the effect of dimensions on the temperature of wires carrying the electric current. The theoretical conclusions of Professor G. Forbes were completely borne out by the results of my experiments; and those now given by Professor Ayrton and Mr. Kilgour agree.

On June 17, 1884, I had the honour of reading a paper, which was wholly occupied with the effect of dimensions, before the Royal Society ('Proc. Roy. Soc.,' No. 232, 1884). In the autumn of the same year I contributed a paper to the British Association on the same subject. These papers (of which the latter was published in 'Nature,' September, 1884) contain a practical experimental investigation of the loss of heat from thin bare wires in air, so far as dimensions are concerned, as well as experimental results as to the effect of coverings of various kinds and different thicknesses.

I have still a large number of results on these subjects hitherto unpublished; and as I have received, from the £4000 Scientific Fund, grants in aid of these researches, I am unwilling that there should be any misunderstanding in this matter.—March 2, 1893.]

† 'Proceedings Royal Society, No. 232, 1884.

‡ 'Phil. Trans.,' 1887.

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have made some progress, particularly in the determination of radiation from incandescent filaments of carbon. The loss of heat from a heated body, however, depends to some extent on the form and dimensions of the body, and it seemed very important to experiment on the emission of heat from bodies differing in form from the wires I had been using, and larger in dimensions. I desire now to give an account of some experiments in this direction.*

In the autumn of 1889 I commenced to experiment on loss of heat from the copper globes used in the experiments of Mr. D. MACFARLANE, 'Roy. Soc. Proc.,' 1872, p. 93. These globes are two exactly equal solid copper spheres, 4 centims. in diameter; one has a smooth copper surface, the other is now thoroughly well electroplated with silver. A considerable number of preliminary experiments were tried, using apparatus similar to that which MACFARLANE employed, and in particular the same enclosure; and then, as my object was to experiment first with full air pressure and afterwards with vacuum more or less complete up to the highest attainable, new and suitable arrangements were made for this purpose.

It is unnecessary here to describe in detail the apparatus used and the mode of experimenting in these preliminary investigations.† It was similar in every respect to that of MACFARLANE. The thermo-junctions were new, however, and were standardized for the purpose of these experiments; the galvanometer arrangements were also fresh and were very carefully made to secure sensitiveness and permanence of the thermometric value of the deflections. The globes had their surfaces prepared in the ways shown in the columns of the table on p. 593; and the metallic enclosure used by MACFARLANE was employed. The headings of the table will, it is hoped, make it self-explanatory.

* The experiments described in what follows were concluded some time ago. I have delayed the giving an account of them in the hopes of finding time and opportunity for obtaining further results; but I have been unable to do so up to the present, and judge it best to publish now the results which I have obtained.

† In connection with these preliminary experiments I wish to acknowledge very cordially the great assistance I received from Mr. JAMES H. GRAY, M.A., B.Sc., at the time of the experiments a young student in the Laboratory, now the holder of the first 1851 Exhibition Scholarship for Glasgow University. To his care and assiduity I am greatly indebted.

TABLE I.—Showing Loss of Heat per square centim., per second, per 1° C. of Excess of Temperature of Cooling Globe above Temperature of Surrounding Enclosure; MACFARLANE'S Enclosure being used, and the Surface of Globe being prepared in various ways as shown in the Headings of the Columns.

Copper globe polished bright. (Mean of six experiments.) Temperature of enclosure approximately 14°·5 C.		Copper globe thinly coated with lamp-black. (Mean of six experiments.) Temperature of enclosure approximately 14°·5 C.		Copper globe polished bright and thinly lacquered. (Mean of six experiments.) Temperature of enclosure approximately 14°·5 C.		Copper globe plated with silver. (Mean of six experiments.) Temperature of enclosure approximately 14°·5 C.	
Temperature of globe.*	Loss of heat per square centim., per second, per 1° C. of excess.	Temperature of globe.	Loss of heat per square centim., per second, per 1° C. of excess.	Temperature of globe.	Loss of heat per square centim., per second, per 1° C. of excess.	Temperature of globe.	Loss of heat per square centim., per second, per 1° C. of excess.
°		° 21	·000278	° 21	·000246	°	
22	·000165	22	·000281	22	·000250		
23	·000170	23	·000284	23	·000254	23	·000174
24	·000174	24	·000287	24	·000257	24	·000178
25	·000178	25	·000290	25	·000260	25	·000182
26	·000181	26	·000293	26	·000263	26	·000185
27	·000184	27	·000296	27	·000265	27	·0001875
28	·000187	28	·000299	28	·000268	28	·000190
29	·000190	29	·0003015	29	·000271	29	·000192
30	·000192	30	·0003045	30	·000273	30	·000194
31	·000194	31	·000307	31	·000276	31	·000196
32	·000196	32	·000310	32	·000278	32	·000198
33	·000198	33	·000313	33	·000280	33	·000200
34	·0001995	34	·000316	34	·0002825	34	·000201
35	·000201	35	·000320	35	·000285	35	·000203
36	·000202	36	·000323	36	·000287	36	·000205
37	·0002035	37	·000326	37	·000290	37	·0002065
38	·000205	38	·000329	38	·000292	38	·000208
39	·0002065	39	·000332	39	·000294	39	·0002095
40	·000207	40	·000335	40	·000297	40	·000211
41	·000208	41	·000338	41	·000299	41	·0002125
		42	·000341	42	·0003015	42	·000214
				43	·000304	43	·000215
				44	·000307	44	·000216
				45	·000309	45	·000218

* N.B.—It is the temperature of the cooling body which is stated in this column, not the excess temperature.

In two cases, the highest polished copper and the sooted globe, the surfaces are similar to those used by MACFARLANE. In the other two cases, "copper polished and finely lacquered" and "copper silvered," the surfaces are new. It is interesting to note that the results for lamp-black differ very appreciably from those obtained by MACFARLANE. All my experiments show that there is difference in radiation from different kinds of lamp-black; though I doubt, considering the way in which "lamp-black radiation" is often referred to, whether the ideas commonly held quite correspond with this view.

On the other hand, the column showing loss of heat from the "copper ball polished bright," agrees, with almost perfect exactness, with the numbers obtained by MACFARLANE under the same circumstances. This confirms, in a very important way, both sets of results. It is quite natural that if both are accurate the two should agree perfectly, because to obtain a well-polished surface of copper again and again in much the same condition is comparatively easy. It was found very difficult, however, to keep the polished surface of copper unoxidized, even for a few hours. Hence, I tried a thinly-lacquered copper surface. The lacquer, of course, alters the condition entirely from that in which the surface is highly polished and unlacquered, but the condition was permanent, and, indeed, it is only in this condition that bright copper is ever met with in the arts or in the laboratory. It might be interesting to try a dull surface of copper lacquered, just because it is the most common of all. The silvered surface was the first I had tried of the kind. It was bright; but no special pains were taken (such as I afterwards took) to get the very highest possible polish. It is worth observing that the bright fresh copper shows a slightly lower rate of loss of heat, probably lower radiation, than the silver-coated globe, while the increase with temperature of the rate of loss is almost the same in these two cases, but very different in the other cases.

In all these cases the air was at the natural pressure of the day. It was not artificially moistened, as was the case in MACFARLANE's experiments, nor artificially dried, as in the case of my own experiments which followed.

The experiments of MACFARLANE, and those just referred to, were made with the copper globes suspended in an enclosure of large dimensions, and at ordinary atmospheric pressure. To make use of a Sprengel vacuum, however, and to compare results obtained with vacuum of various degrees of pressure and at full air pressure, it is necessary, according to the theory of CROOKES' results, to have an enclosure of moderate dimensions, so that the condition may be reached in which the length of free path of the gas molecules becomes comparable with distances, from point to point, of portions of the enclosure.

For the reasons stated in my former paper,* I also considered it essential that the enclosure should be metallic. I considered whether a glass globe lined inside with

* 'Phil. Trans.,' 1887, p. 444. Uncertainty, with a substance so badly conducting and so little diathermanous as glass, as to the temperature of the inner skin of such an enclosure.

metal, by first silvering and subsequently depositing copper on the silver, would be satisfactory; but finally determined, in spite of the difficulty of joining a metal enclosure to the Sprengel pump, to use a hollow copper sphere.

At the centre of the hollow sphere, the heated globe is suspended and allowed to cool, the enclosure being kept immersed in water at a known temperature. The temperature of the cooling globe is read off at equal intervals of time by means of a thermo-electric junction, and from these readings the rate of cooling, and the absolute loss of heat per unit of cooling surface, per unit difference of temperatures of surface and surroundings, per unit of time, are calculated.

The details of the arrangement will be understood from the following description, by reference to figs. 1 and 2, Plate 17. In fig. 1, *aa* is the spherical enclosure. It is a copper spherical shell, 10 centims. in internal diameter, constructed like the Magdebourg hemispheres, but surmounted by a brass tube, *tt*, which is brazed to the upper opening of the copper shell. At the top of the brass tube is a groove, made by brazing on a small piece of wider brass tube; and into this groove the extremity of the glass tube, *gg*, is inserted. The extremity of the glass tube is prepared by grinding to fit the socket; and it is joined in, air-tight, at the last moment, when all the other preparations are complete, by means of *siegelwachs*, a soft German cement which I have found invaluable for these purposes, and have already mentioned in my former paper.* By means of a narrow glass tube, *ee*, the enclosure is attached to the Sprengel pump, being hermetically sealed to it in the usual way. An outer tube of brass, TT, forms a water-jacket to the tube, *tt*; and, by means of a suitably-arranged tube, water is kept flowing round the *siegelwachs* joint. The copper sphere is suspended in the enclosure by one of the wires of the thermo-junction.

For the thermo-junction I employ platinoid and platinum. The latter is used because it is to be sealed into the glass tube *gg*, as is shown at *s*; and no wire is known at present, except platinum, or an alloy of platinum, which, when sealed into glass, will make a joint sufficiently tight for a Sprengel vacuum. I have found platinum and platinoid to give a very satisfactory thermo-junction; and, along with Mr. A. TANAKADATE, have published a paper on the thermo-electric position of platinoid alloy.†

The platinoid wire is passed through the brass tube *tt* at *p*, and soldered in its place. It is, therefore, not insulated; and it bears the weight of the copper sphere. The platinum wire bears no weight. It is insulated all the way up through the brass tube by being encased in a very thin tube or sleeve of glass, which is slipped over it. The platinum wire has a small length of spiral at the top to take up any slack that may exist (as it is not desirable, for glass-blowing reasons, to cut off the platinum wire to the exact length), and to avoid any pull on the glass at the place where the wire is sealed into the wall of the tube. The platinoid wire is cut by trial to the proper length, so that, when the weight of the copper globe is on it, the

* 'Phil. Trans.,' A., 1887, p. 433.

† 'Roy. Soc. Proc.,' vol. 46, 1889, p. 286.

globe shall hang at the centre of the spherical enclosure : the ends of the platinum and platinoid are then brought together and inserted into a hole drilled in a copper plug, and are silver-soldered to each other and into the hole. This copper plug is firmly fitted by screwing into a hole, which is carefully cut in each of the copper globes.

The operations just described are, of course, carried out at the commencement of a series of experiments, the copper plug being once for all attached to the thermo-junction. The globes can be attached or detached at pleasure for the purpose of having the surfaces polished, sooted, or otherwise treated. When a globe has been hung in its place for experiment, the lower half of the spherical enclosure is brought up, pressed firmly to the upper half, and soldered there.

It has already been mentioned that, during the cooling of the heated globe, the enclosure is immersed in a bath of cold water, which is kept at constant temperature. An ordinary chemical water-bath of copper is used for this purpose. It is shown surrounding the enclosure *aa*. One of the thermo-junctions being, as has been described, at or near the centre of the cooling globe, the second junction is soldered to a small lump of copper, which is tied to the bulb of a thermometer and immersed in the water-bath. The platinum wire is made sufficiently long to pass from the globe up through the brass tube, out through the side of the glass tube at *s*, and down to the water-bath, where its extremity joins the second platinoid wire at *m*, the small lump of copper just mentioned. The two platinoid wires, *pd*, one coming from the centre of the cooling globe, and the other from the junction *m*, are carried forward to *o*, where they are soldered to the copper wire electrodes of a reflecting galvanometer to be spoken of immediately. It is necessary that no electromotive force, due to differences of temperature, should arise at these joinings. These joinings are, therefore, wrapped in tissue paper for insulation ; and then a large quantity of cotton wool, *o*, is wrapped round the whole and along the wires for some distance. The protection and equable distribution of temperature is so perfect that the cotton wool bundle may be handled freely with warmed hands without the galvanometer being in any way disturbed.

The galvanometer used in these experiments is a THOMSON's reflecting galvanometer. It has a very excellent coil, 0·9 ohm resistance, specially wound for me for use with thermo-junctions, by JAMES WHITE, Glasgow. I use a Steinheil plane parallel mirror, hung by a spider line instead of silk fibre, in a THOMSON's dead-beat plug. The comfort and satisfaction of a spider line for purposes where the galvanometer deflection is to be read, and not merely a zero noted, or where a steady unchanging zero is desired, are almost indescribable.*

Placed opposite the mirror is a Steinheil telescope and a scale. Every part of the galvanometer apparatus is screwed down on a wooden bracket fixed to the stone wall of my experimenting room, and the field magnets are fixed down firmly with cement. I use a small weak "zero magnet," with its magnetic axis perpendicular to

* J. T. BOTTOMLEY and A. TANAKADATE. 'Roy. Soc. Proc.,' vol. 46, 1889, p. 286.

the plane of the mirror, for the purpose of bringing the zero of the scale to the cross wire of the telescope. This is not often required, but occasionally, when the sun shines strongly on the iron frames of the Glasgow University windows, the zero may be pulled round by a few divisions; and, occasionally, also a movement of some iron in the laboratory slightly displaces the zero. The "zero magnet" being moved slightly, with its axis along a line perpendicular to the plane of the mirror, adjusts the zero without altering the field of force. I find it an excellent plan to put a little "soft wax"* on the zero magnet, and press it on to the table in front of the galvanometer shelf. By cautiously pressing on the magnet it may be made to slide very slowly, with viscous yielding of the wax, for the purpose of setting the zero of the galvanometer.

I have mentioned already that I have made, with the assistance of my friend, Mr. A. TANAKADATE, a determination of the thermo-electric value of a platinum-platinoid couple. We considered it necessary, however, to make special determinations of the values of the actual couples used in these experiments, and this was done each time a fresh soldering of the platinoid and platinum was made. Platinum wire is so variable in its qualities that it is not safe to assume that the physical properties of different portions of the same hank even are the same.†

The standardizing of the thermo-junction is performed with the circuit precisely as it is to be used during the radiation experiments, with every joint soldered, and with the field of force of the galvanometer suitably adjusted and rendered unchangeable, as described above. Along with the thermo-electric wires, the leads, and the galvanometer, a platinoid resistance of about 12 ohms was introduced into the circuit. The sensitiveness of the arrangement was quite sufficient to permit of this being done; and it was of great advantage to introduce this added resistance, as it rendered insensible small variations of resistance (due to temperature) in the long platinum thermo-electric wire, which in early experiments gave rise to curious and embarrassing variations in the galvanometer readings.

From the standardizing, or calibration, as it is often called, of the thermo-electric circuit in use, a formula is obtained, from which is calculated, from the "*corrected* deflection"‡ of the galvanometer, the difference of temperatures of the two junctions in terms of centigrade temperatures on the air thermometer.§ Every temperature

* The material used by chemists for making diachylon sticking plaster answers admirably.

† The best makers will not, I believe, guarantee to supply *uniform* platinum, in the physical sense of the word uniform, knowing that it cannot be done, at present at all events.

‡ The deflection is read off on a plane scale, and is *corrected* for the fact that the angle turned through by the light ray is twice the angle of deflection of the magnet and mirror. In order to apply this correction with facility, a table of numbers was constructed. The correction may now, however, be easily applied by the use of the recently published "Reductions-Tabellen zur GAUSS-POGGENDORFFSCHEN Spiegelablesung," by Dr. PAUL CZERMAK. J. SPRINGER, Berlin.

§ Sir WILLIAM THOMSON'S article "Heat," 'Encyc. Brit.,' and J. T. BOTTOMLEY, 'Proc. Roy. Soc., Edinburgh,' 1888, and 'Phil. Mag.,' August, 1888.

referred to in this paper is thus temperature on REGNAULT'S "normal constant volume" air thermometer. All the mercury-in-glass thermometers employed for subsidiary purposes were also compared directly with the air thermometer, and the suitable corrections applied wherever they were sensible.

One part of the arrangement of the thermo-electric circuit still remains to be mentioned. By far the most satisfactory way of experimenting for purposes like those under consideration is to take readings (in the way to be described immediately) of the galvanometer deflection on each side of the middle, or zero, position, at equal intervals of time, instead of reading only the deflection on one side, as is often done. Accordingly, I inserted in the circuit a mercury reverser, constructed with care to avoid the introduction of small uncertainties as to resistance. The use of this piece of apparatus also enabled me from time to time to apply to the galvanometer and its leading wires a very small known current from a standard cell, in order to assure myself of the constancy of the magnetic field.

I may now explain the mode of experimenting. The value of the thermo-junction having been determined, and the copper globe screwed on to the plug, and the apparatus, which is supported by clips on a retort stand, having been adjusted so that the globe shall hang in the centre of the enclosure, the lower half of the enclosure is brought up and held pressed against the upper half by means of temporary clips. Owing to the edges of the flange having been well prepared by grinding, and by having a very thin layer of solder or tinning applied to them, the parts fit together very exactly; and in this position they are now soldered together, with the help of a soldering bolt and a Bunsen burner. Resin is used for the soldering, except just at the end of the operation, when a little soldering fluid is applied round the outer edges to make sure of perfect filling up of the minutest holes. The soldering up of these edges in such a way as to be tight for the Sprengel vacuum is a very troublesome operation, and one which requires the greatest care.

When the soldering is finished, the joint between the brass tube and the glass tube is made complete by means of *siegelwachs;* and finally the apparatus is connected to the SPRENGEL pump by means of a portable blowpipe. The intention, generally speaking, in these experiments, was to determine the loss of heat from surfaces prepared in various ways, both with air present (and later with other gases), and also in more or less complete vacuum. Sometimes the cooling in vacuum was tried first, and the cooling in air afterwards; sometimes the order was reversed. Sometimes the cooling in air was determined, then in vacuum, and then in air again.

This being understood, I may say that everything being prepared as described above, water was now poured into the water bath; a flame was applied beneath, and the boiling water communicated heat to the enclosure, and gradually to the suspended ball within. The galvanometer was watched and the time roughly noted to show the progress. I have not, however, made use hitherto of these observations on speed of heating.

With full air present, the process of heating was not inconveniently slow. With

vacuum it was excessively tedious, several hours being required for the heating. The process was carried on till the thermo-junction showed that the boiling-point of water had been very nearly reached by the suspended globe.

Then, preparations having previously been made, the lamp was extinguished; the boiling water was very quickly run out with the help of siphons, and cold water run in, in place of it. A supply of cold water, at very nearly the temperature of the laboratory, was kept in readiness for this process, and a plentiful supply was run through the water-bath until all the surroundings had come to a steady temperature. Neighbouring parts of the table, retort stands, &c., were often cooled by wetting to obtain subsequent steadiness of temperature. In my experiments I had the advantage of the able assistance of Mr. A. TANAKADATE. I cannot sufficiently express my indebtedness to him, either for his dexterous manual aid, or for his many ingenious and valuable suggestions during the progress of the work. Single-handed, some of the work would have been impossible; and there are few, even of trained experimenters, who could render such help as he gave me.

As soon as the galvanometer could be read, after the cold water had been applied, regular observation commenced, and the method of observation was as follows:—It was found that an observation every 5 minutes was sufficient. (In experiments at higher temperatures, described below, the intervals were $2\frac{1}{2}$ minutes.) A chronometer, beating half-seconds, stood close by; and by listening to the beats, as in astronomical observations, the galvanometer reading was taken at 10 seconds before the exact minute. The reverser was instantly shifted, and the first reading then recorded. By the time that 4 or 5 seconds after the exact minute was reached, the observer was once more ready, looking through the telescope and counting the seconds, and the vibrations of the galvanometer (well damped*) had long before completely subsided; and, at exactly 10 seconds after the minute, the observer was able to take the second reading. The sum of these readings, to right and left of the galvanometer zero, was taken to represent the temperature at the exact minute noted; and the mean of the pairs of readings, if it agreed with the known zero of the galvanometer, afforded a check, and showed (1) that no mistake had been made in noting; (2) that the galvanometer needle was *not* on the swing at the second reading; and (3) that there was no disturbance from the distant dynamo or from the moving about of magnets in the laboratory.

The readings were at once plotted down on squared paper; and at first this plotting revealed little faults in observation, shakiness of apparatus, change of resistance in the leads, &c., all of which faults were soon eliminated. The curves then became so perfectly regular as to leave nothing to be desired.

Before giving an account of the results obtained, I will briefly describe a modified

* Damped by the air damping of a Thomson dead-beat galvanometer plug. It seems strange that the Thomson dead-beat plug is so little known to instrument makers, and to users of the reflecting galvanometer. It has been in use in all the galvanometers supplied by JAMES WHITE for about twenty-five years, and no instrument of the kind which is not dead-beat would be tolerated in the Glasgow laboratory.

apparatus which I found it necessary to use for temperatures higher than the boiling point of water.

When I had finished a considerable number of determinations, carried on precisely as described above, with boiling water for the superior temperature, I attempted to get to a temperature of 200° C. or 250° C. I found, however, that the soldered joint would not bear a temperature of anything like 200° when the Sprengel vacuum was applied. The solder became crystalline and brittle and leaky below 150°. The solder used was good ordinary solder; and I think that slight leakiness (very perceptible to the Sprengel) manifested itself at a temperature at which the solder was for ordinary purposes quite good and sound. I am not sure, however; and it is quite possible that by trials I might have got a better solder by using less tin in the composition. I did not delay with such trials, but preferred to alter the enclosure to the form shown in fig. 2. *aa* is a copper shell nearly spherical, but with a wide mouth and neck. The shell is beaten into shape. The neck is carefully brazed on, and the parts round the brazing finished by beating. The neck has a flange with the upper edge ground flat. The whole is very thick and solid, and an outer shell of copper was subsequently added, squeezed into shape and fitting loosely round the other for the purpose of equalizing the heating (and cooling) of the enclosure. The brass tube *tt* and outer water-jacket are also shown in the diagram; and it will be seen that, to close the copper shell, the whole of this piece is brought down on the flange at the neck of the shell; and the soldering is made at this joint. This construction was found to be in every way an improvement on that used at first. The only little trouble being the lowering in of the copper globe without touching the sides, and the making the measurements necessary for bringing the copper globe to the centre of the enclosure. These troubles are scarcely worth mentioning, and the new arrangement gave much greater freedom for working at the globe while it was hanging on the wire before being lowered into its place.

In the arrangement just described it will be seen that the soldering is kept cool by the proximity of the cold water in the water-jacket; and this can be done even although the temperature of the shell enclosure is very high. In practice a stream of cold water was kept running through the water-jacket during the heating process described above, and for heating the enclosure a Bunsen burner or, in the latest experiments, one of FLETCHER's high-power burners was applied direct to the bottom of the copper shell.* When the temperature of the globe inside was found,

* A curious phenomenon, which requires farther examination, was noticed during the heating of the globe described in the text. I found it impossible to exhaust the enclosure with the Sprengel pump while the flame was applied; but as soon as the flame was removed I could exhaust the enclosure perfectly. Whether the copper shell became porous at the high temperature (just visible redness) and thus admitted gas enough to prevent my obtaining a vacuum; or whether some minute crack or opening was made at some of the joints owing to the great variation of temperature from point to point of the shell, I am unable to say at present. The leaking ceased, however, as soon as the heating ceased. This leak-

by means of the galvanometer, to have risen to the desired point, the flame was withdrawn, and the cold water, at the temperature of the laboratory, applied as before. It was also found advisable for steadiness of temperature to stop the current of cold water through the water-jacket, and to warm up the water in the water-jacket (which, while the flow continued, was kept at the temperature of the public supply) to be roughly the same as the air temperature of the laboratory.

The Sprengel pump used for exhausting the enclosure and the duplicated MacLeod gauge used for determining the residual pressure are fully described, and certain difficulties and uncertainties are pointed out, in my former paper,* and need not be further spoken of here.

For the purpose of calculation two pieces of numerical information are still required. One is the capacity for heat of the copper globes; and the other, the cooling surface.

The capacity for heat was determined very carefully for these particular globes by Mr. D. MACFARLANE in 1872; and my determination has not sensibly varied from his result. I propose, however, to make a fresh determination of this constant as soon as possible.

The surface of the globe is obtained by careful measurement with calipers. But it will be observed that, besides loss of heat by radiation, there is also a loss by conduction along the two thermo-electric wires. When a sooted globe is cooling in full air pressure the amount of heat conducted along the thin wires used is very small in comparison with that lost by radiation and convection. But when the globe is cooling in vacuum or, at any rate, when a highly polished silvered globe is cooling in vacuum, the heat lost by the wires is by no means negligible in calculating the value of the surface emissivity.

In making use of the results obtained up to the present time I have used for calculation an equation of the form

$$- c \frac{dv}{dt} = e\mathrm{S}\,(v - v_0),$$

ing gave rise to a most disturbing difficulty at the commencement of the corresponding vacuum series; because it was desirable to commence the observations on cooling as soon as the cooling commenced, but it was not till ten or fifteen minutes later that a proper vacuum could be secured, even with the most vigorous use of the pump.

* 'Phil. Trans.,' 1887, pp. 444–446. There is just one remark which it seems worth while to make, on account of repeated misunderstanding, namely, that the MacLeod gauge does not measure (nor profess to measure) pressure in the pump of any gas collapsible by pressure, for any reason whatever, at the existing temperature of the gauge. Thus it does not measure the pressure of mercury vapour which may be present in the so-called vacuum, nor the pressure of vapour of water, for example. Nor does it even measure the pressure of any gas which may with the help of a small pressure be caused to collapse against the walls of the tube, or by absorption into, or on, the mercury used in the gauge. I have shown the possibility of this last-named occurrence in a paper on an "Improved Mercury Pump," British Association, 1886.

where v is the temperature of the cooling globe, v_0 that of the enclosure, c the capacity for heat of the globe, and S its surface, while e is a co-efficient which has been called the *emissivity*. It is easy to show by an application of FOURIER's equations that, to a first approximation, sufficient for my present purpose, the loss by conduction by the wires may be allowed for by adding to S, or I should rather say to eS, a small quantity depending on the diameter, conductivity, and emissivity of the wire, the length of the wires being so great that no heat passes away by conduction at the ends. The numerical value of the quantity to be added to eS I have not yet determined exactly; but I have made an estimate of it and applied it as a correction in obtaining the results given below.

Another interesting point requiring consideration is the following:—The arrangements explained above give approximately the temperature at or near the centre of the copper globes. Those of Mr. D. MACFARLANE did the same in the case of his experiments. In a criticism of MACFARLANE's results, by M. CORNU, 'Journal de Physique,' December, 1873, an objection was taken to the thermo-electric method as compared with the method used by DULONG and PETIT, where the bulb of a thermometer was the cooling body, on the ground that the temperature of the centre may be different from that of the surface. This objection was completely answered by numerical calculations founded on the dimensions and conductivities of the globes in question; and, in an interesting appendix the whole question is discussed in the 'Royal Society Proceedings,' June 10, 1875.*

With these explanatory remarks I proceed to a brief statement of the results obtained, but I wish it to be clearly understood that I do not yet consider myself entitled to draw general conclusions as to laws of radiation from the observations completed and calculated down to the present date. Theories of radiation have, in recent years, been discussed by STEFAN,† H. F. WEBER,‡ FERREL,§ and others; but it appears to me that the main difficulty of making any satisfactory theory is the want of experimental data on which to found a theory, and by which to test it. I venture to think that the data required are still far from sufficient; and, indeed, this is admitted and pointed out as a difficulty by the most recent of these writers, the late Mr. FERREL.

I have experimented solely on the copper globes with their surfaces prepared by sooting and silvering. The sooting was done very carefully with a gas flame or with paper or cotton waste impregnated with powdered resin, or powdered camphor, or moistened with turpentine. The endeavour was to put on a thoroughly continuous but thin coating of lamp-black, and thus to obtain what I presume we may take to be a surface giving maximum of radiation. On the other hand the silvered

* Republished in 'Lord KELVIN's Collected Papers,' vol. 3, p. 245.

† STEFAN, 'Berichte der Wiener Akademie,' vol. 79, 1879.

‡ Professor H. F. WEBER, 'Akademie der Wissenschaften zu Berlin.' Berlin, 1888.

§ W. FERREL, 'American Journal of Science,' July, 1889, and February, 1890.

globe was polished to the highest degree possible. The polishing was done very frequently, and with special care, by the kindness of Mr. THOMAS SMITH, Electro-plater, Glasgow. After polishing the globe was wrapped in tissue paper and kept, surrounded by a very large quantity of cotton wool, in a box until the last moment before it was screwed on to the plug by which it was suspended. It was never touched by the bare hands, being held in its tissue-paper cover while the operation of screwing on was performed. The surface was so smooth and slippery that it was difficult to keep fast hold of the globe while it was being screwed on; and when suspended it presented the most beautiful mirror surface I have ever seen.

I am supposing that the minimum of radiation is had from such a silver surface as has just been described; and it was extremely interesting to find that a marked effect, in reducing the amount of radiation in vacuum from the silvered globe, was produced by efforts to attain the highest possible degree of polish.

In calculating the emissivity from the recorded deflections of the galvanometer, the first process was to obtain the "corrected deflection"* of the galvanometer. Secondly, a correction is applied to the numbers for the fact that the deflection produced by the thermo-junction is not in simple proportion to the difference of temperatures (defined by the air thermometer) between the two junctions.† Thus a series of numbers is obtained corresponding to the successive times noted, the intervals being 5 minutes or $2\frac{1}{2}$ minutes, as has already been mentioned. From these numbers I have calculated the value of the "emissivity," or quantity of heat lost per second, per square centimetre of surface, per degree centigrade of difference of temperatures of surface and surroundings.

Let c be the capacity for heat of the cooling globe, S the surface, and $(v - v_0)$ the difference of temperatures of source and surroundings at any time t. Then in the formula

$$- c \frac{dv}{dt} = e\text{S} (v - v_0),$$

the coefficient e corresponds with the definition of "emissivity" just given; and this formula is commonly taken to be a representation of the "law of cooling," whether in air or in any other gas, or in vacuum, the range of temperatures dealt with being moderate. The numerical value of e, however, depends on the circumstances under which the cooling takes place; and, when air is present, on the dimensions and shape of the cooling body. Important theories, to which I have already alluded, have been from time to time put forward as to the form, practically speaking, of e as a function of v for the case of pure radiation.

One way, however, of dealing with the matter, from an experimental point of view,

* See note p. 599.

† For example, one of the formulas, with numerical coefficients actually used, was as follows:—

$$t = \frac{1}{3\cdot38} \delta - \cdot000086 \delta^2,$$

where t is the difference of temperatures, and δ the "corrected deflection" of the galvanometer.

is to solve the equation above as though e were constant—which is, of course, approximately true if the difference of temperatures is small—and then to determine, numerically, the value of e at different places in the temperature scale, taking an exact account of the circumstances. When a sufficiently large number of such values have been obtained a basis for a proper theory will have been laid in a logical way. This is practically what I am endeavouring to do.

Taking, then, the equation

$$- c \frac{dv}{dt} = e\mathrm{S} (v - v_0),$$

and its solution

$$\log_\epsilon \frac{v' - v_0}{v - v_0} = \frac{e\mathrm{S}}{c} t,$$

where v' is the temperature of the cooling globe when $t = 0$, we have for the numerical calculation of e

$$e = \frac{c}{\mathrm{S}t} \{\log_\epsilon (v' - v_0) - \log_\epsilon (v - v_0)\},$$

or, using common logarithms,

$$e = \frac{\mathrm{M}c}{\mathrm{S}.t} \{\log (v' - v_0) - \log (v - v_0)\}.$$

Here $t = 300$, the intervals of time used in my experiments being 5 minutes; $c = 28{\cdot}31$ and $\mathrm{S} = 50{\cdot}26$, with an addition which, for the present, I roughly estimate at about 0·6 per cent.—the correction applied for the carrying away of heat by the conducting wires. This I calculate, assuming that the emissivity of the surface of the conducting wires is much the same as that of a *tolerably clean* silvered surface.

The following tables and curves (Plate 18) show most of the numerical results obtained. The tables are made self-explanatory, as far as possible, by means of suitable headings; and the curves which I have drawn corresponding to most of the series are dated, and the multiplier is given for reducing the ordinates to C.G.S. measure. The curves were drawn chiefly for my own satisfaction; but there are one or two matters to which I may be permitted to call attention.

In the first place, it will be seen from the curves that in each series the results are in very exact accordance. Certainly they are more so than I ventured to hope when I commenced putting them down. The difficulties surrounding experiments of this kind are really very great, if anything more than rough approximation is aimed at; and, unfortunately, uncertainties connected with the temperature measurement easily creep in, and are difficult to detect till it is too late to remedy them, or even to determine the limits of possible error.

The different series of determinations represented by the several curves are also in agreement, and some of them, as, for instance, those of March 7 and 10 (silvered globe in air), and those of October 29, 1889, and April 2 and 8 (sooted globe in vacuum), are quite unexpectedly close to each other. There are differences between

the absolute positions of the curves, but most of these depend on some differences of circumstances. The only case which I cannot explain is that of the silvered globe in air, November 6. I do not understand why the loss of heat was so small in an experiment which bears every sign of being thoroughly good. I have no note (such as would certainly have been made) of any change of galvanometer zero (indicating possible alteration of magnetic field); nor was there alteration of the apparatus of any kind.* Extreme dryness of the air is a possible explanation.

A feature, which seems to me remarkable, about the curves representing these late experiments is that none of them show any sign of emissivity, as it were, accelerated with increase of temperature, that is to say, having an increasing rate of increase of emissivity with increasing temperature, but rather the reverse. The curves are all slightly concave towards the axis of temperatures. It is, however, well ascertained, particularly from the experiments of SCHLEIERMACHER and myself, that, at all events, in the case of *wires* losing heat there is a very rapid acceleration or increase of rate of increase of *radiation* with increase of temperature. This is most easily seen by looking at the curves of my former paper, 'Phil. Trans.,' 1887, A, Plate 25. Looking at the matter in this light also, it will be seen from those curves that the curvature (proportionate acceleration) is greatly masked when more than $\frac{1}{4}$ millim. or $\frac{1}{2}$ millim. of air pressure is present, and that the lower the air pressure the more striking is the convexity towards the axis of temperatures in that series of curves.

Referring now particularly to the tables accompanying this paper. Table I. has been already explained (pp. 591, 592, 593). Table II. shows the results of four nearly successive days of experimenting in air and in vacuum on a globe with two different coatings of lamp-black. In the first pair the coating was much more dense than in the second pair. In the latter case the globe was passed quickly a few times through the gas flame of my blowpipe table. In this way a coating of the finest texture was obtained; the colour of the copper globe was distinguishable through it. I had a reason for wishing to keep the coating very thin in connection with experiments on incandescent wires which I am making.

Table III. shows the results of similar experiments on a silvered globe.

Both these sets of experiments were made with the apparatus of fig. 1; and I believe that in neither was the interior of the enclosure sooted in the way which was afterwards done.

Tables IV. and V. show the results of experiments on the sooted globe in air and in vacuum, and on the silvered globe in air at very high temperatures. These experiments were made in apparatus of fig. 2, Plate 17, and were most satisfactory in every way.

I come lastly to Table VI. It was in the hope of finding time (three months at least) for repeating and extending these experiments that I have delayed the publi-

* To imagine a cause for unexpected excessive loss would be easy; but it is not easy to account for the globe retaining its heat.

cation of this paper. The operation of soldering up the enclosure so thoroughly as to withstand the high temperature required for heating up the hanging globe and to remain perfectly air-tight for the Sprengel pump was at all times difficult. But when the enclosure contained the silvered globe, in the highest state of polish, it became almost impossible to perform the operation without tarnishing the globe. The experiment of April 14 was satisfactory, except for the somewhat variable state of the vacuum; which was, however, very high throughout. The numbers quoted show the effect of this variableness. A new experiment was tried on April 17; but the result was less satisfactory; the globe turned out to be tarnished when the enclosure was opened; and the rapidity of loss of heat corresponds.

Unfortunately, the time at which these experiments were tried was the very end of the Glasgow winter session. Indeed at this time I had already become involved in the heavy University work belonging to this period of the year. Before we could resume experimenting, my friend Mr. TANAKADATE was obliged to leave Glasgow for Berlin; and I have not since then found the time and circumstances necessary for repeating these last experiments with an enclosure slightly modified to make the soldering more easy and more sure.

TABLE II.—Copper globe sooted. (Apparatus fig. 1.)

October 22, 1889. Vacuum. Pressure $\frac{1}{1\cdot 3}$ M.* Temperature of bath surrounding enclosure, 17°·5 C.		October 24, 1889. Air. Pressure 75 centims. Temperature of bath surrounding enclosure, 16°·4 C.	
Excess of temperature of globe above temperature of surroundings.	Loss of heat per sq. centim. per second per 1° C. of excess.	Excess of temperature of globe above temperature of surroundings.	Loss of heat per sq. centim. per second per 1° C. of excess.
79°·6	$1\cdot37 \times 10^{-4}$	72°·1	$2\cdot76 \times 10^{-4}$
74·0	1·36 ,,	62·3	2·69 ,,
68·9	1·34 ,,	54·1	2·62 ,,
64·2	1·32	47·2	2·55
59·9	1·30	41·3	2·48
55·9	1·29	36·2	2·41
52·2	1·27	31·9	2·34
48·8	1·25	28·2	2·27
45·7	1·23	25·1	2·20
42·8	1·22	22·4	2·13
40·1	1·20	20·0	2·06
37·7	1·18	18·0	1·99
35·4	1·16	16·2	1·92
31·3	1·13		
27·8	1·09		
24·8	1·06		
22·2	1·02		
20·0	0·989		
18·0	0·954		

* M denotes a pressure equal to one-millionth of an atmosphere. The measurement, however, is not of the TOTAL pressure in the enclosure; but only of the pressure of the gas *not collapsible* in the MacLeod gauge.

TABLE II. (continued).—Copper globe very finely sooted.

October 25, 1889. Air. Pressure 76 centims. Temperature of bath surrounding enclosure, 17° C.		October 29, 1889. Vacuum. Pressure $\frac{1}{2\cdot 2}$ M. Temperature of bath surrounding enclosure, 15°·3 C.	
Excess of temperature of globe above temperature of surroundings.	Loss of heat per sq. centim. per second per 1° C. of excess.	Excess of temperature of globe above temperature of surroundings.	Loss of heat per sq. centim. per second per 1° C. of excess.
76°·1	$2\cdot80 \times 10^{-4}$	83°·6	$1\cdot25 \times 10^{-4}$
68·25	2·73	78·2	1·23
59·1	2·65	73·3	1·21
51·6	2·57	68·7	1·19
45·1	2·50	64·6	1·17
39·75	2·42	60·7	1·15
34·8	2·34	57·1	1·14
30·7	2·26	53·9	1·12
27·2	2·19	50·7	1·10
24·2	2·11	47·8	1·08
21·6	2·03	45·13	1·06
19·6	1·95	42·7	1·05
18·1	1·88	40·4	1·03
		36·3	0·99
		32·7	0·95

It will be of interest, in order to observe the way in which the cooling proceeds, to remark that the excess temperatures set down in the temperature columns are practically those observed at the ends of successive intervals of five minutes.

TABLE III.—Copper globe, silvered and polished. (Apparatus, fig. 1.)

November 4, 1889. Vacuum. Pressure $\frac{1}{3\cdot04}$ M. Temperature of bath surrounding enclosure, 14°·4 C.		November 5, 1889. Vacuum, and most thorough drying since yesterday. Pressure $\frac{1}{2\cdot5}$ M. Temperature of bath surrounding enclosure, 15°·6 C.		November 6, 1889. Air, very dry, admitted in full pressure to enclosure of yesterday. Barometer not noted. Temperature of bath surrounding enclosure, 15°·7 C.	
Excess of temperature of globe above temperature of surroundings.	Loss of heat per sq. centim. per second per 1° C. of excess.	Excess of temperature of globe above temperature of surroundings.	Loss of heat per sq. centim. per second per 1° C. of excess.	Excess of temperature of globe above temperature of surroundings.	Loss of heat per sq. centim. per second per 1° C. of excess.
67°·4	$4\cdot53 \times 10^{-5}$ *	72°·1	$3\cdot54 \times 10^{-5}$	66°·8	$1\cdot75 \times 10^{-4}$
65·6	4·50 *	70·8	3·52	61·2	1·72
64·0	4·50 *	69·5	3·50	55·9	1·68
62·5	4·44	68·2	3·47	51·2	1·65
59·7	4·38	66·9	3·45	46·9	1·61
57·0	4·32	65·7	3·44	43·1	1·58
54·2	4·25	64·5	3·42	39·7	1·54
52·0	4·20	63·4	3·40	36·6	1·51
49·1	4·12	62·2	3·38	33·8	1·47
46·6	4·05	61·1	3·36	31·25	1·44
45·6	4·02	59·0	3·32	29·0	1·41
43·7	3·96	58·0	3·30	26·9	1·37
41·9	3·90	56·0	3·26	25·0	1·34
40·2	3·84	54·1	3·22		
36·4	3·69	53·2	3·20		
34·4	3·60	51·4	3·16		
		49·7	3·12		
		48·1	3·08		
		46·6	3·04		
		45·1	3·00		
		43·7	2·96		

* Vacuum not satisfactory at commencement. Improved to amount stated, $\frac{1}{3\cdot04}$ M, by working pump; but somewhat variable all through. Probable cause, want of perfect drying. This would not be indicated as pressure by the MacLeod gauge.

TABLE IV.—Copper globe sooted. (Apparatus, fig. 2.)

March 29, 1890. Globe freshly sooted, very thin coat, but very complete and uniform. Air. Pressure 75·6 centims. Temperature of bath surrounding enclosure 17°·5 C.		April 8, 1890. Vacuum. Pressure $\frac{1}{5\cdot1}$ M. Temperature of bath surrounding enclosure 15° C.		April 10, 1890. Dry air admitted to enclosure of April 8. Pressure 75·5 centims. Temperature of bath surrounding enclosure 15°·5 C.	
Excess of temperature of globe above temperature of surroundings.	Loss of heat per sq. centim. per second per 1° C. of excess.	Excess of temperature of globe above temperature of surroundings.	Average loss of heat per sq. centim. per second per 1° C. of excess, between the temperatures indicated in preceding column.	Excess of temperature of globe above temperature of surroundings.	Average loss of heat per sq. centim. per second per 1° C. of excess, between the temperatures indicated in preceding column.
224°·0	$3\cdot92 \times 10^{-4}$	230°·4		250°·3	
189·9	3·82		$2\cdot23 \times 10^{-4}$		$4\cdot44 \times 10^{-4}$
155·5	3·68	204·6		217·3	
128·2	3·55		2·00		4·26
106·5	3·42	184·0		189·6	
89·1	3·29		1·78		4·12
75·0	3·16	152·2		173·7	
63·6	3·03		1·63		3·81
54·3	2·90	139·5		157·0	
46·7	2·77		1·52		3·70
40·4	2·64	128·7		142·2	
35·3	2·51		1·44		3·70
31·0	2·38	119·2		126·4	
			1·42		3·63
		110·5		116·9	
			1·36		3·55
		102·8		106·4	
			1·27		3·48
		96·1		89·25	
			1·29		3·23
		89·7		81·9	
			1·20		3 02
		84·1		58·5	
			1·22		2·98
		78·8		49·9	
			1·20		2·86
		74·0		42·9	
			1·26		
		69·2			
			1·15		
		65·0			
			1·07		
		61·4			

TABLE V.—Copper globe silvered and highly polished. (Apparatus, fig. 2.)

March 7, 1890. Air. Pressure 75·8 centims. Temperature of bath surrounding enclosure, 12° C.		March 10, 1890. Air. Pressure 76 centims. Temperature of bath surrounding enclosure, 13° C.	
Excess of temperature of globe above temperature of surroundings.	Loss of heat per sq. centim. per second per 1° C. of excess.	Excess of temperature of globe above temperature of surroundings.	Loss of heat per sq. centim. per second per 1° C. of excess.
°		°	
214·9	$2·53 \times 10^{-4}$	231·3	$2·55 \times 10^{-4}$
188·7	2·49	221·1	2·52
165·5	2·45	193·4	2·48
145·4	2·41	169·5	2·44
128·0	2·37	148·9	2·40
113·0	2·33	131·2	2·35
99·9	2·29	118·5	2·31
88·5	2·25	102·4	2·27
78·6	2·21	90·8	2·23
69·8	2·17	80·7	2·18
62·3	2·13	71·9	2·14
55·8	2·09	64·2	2·10
50·0	2·05	57·4	2·06
44·9	2·00	51·5	2·01
40·4	1·96	46·3	1·97
36·4	1·92	41·7	1·93
32·8	1·88	37·5	1·89

TABLE VI.—Copper globe silvered and very highly polished for experiment of April 14; believed to be tarnished during experiment of April 17. (Apparatus, fig. 2.)

April 14, 1890. Vacuum. Pressure at commencement about 10 M or 12 M, coming down in 25 minutes to less than $\frac{1}{10}$ M, and kept so, though somewhat variable, by vigorous working of Sprengel. Temperature of bath surrounding enclosure, 13°·3 C.		April 17. Vacuum. Pressure 2 M. Considerably more at commencement. Temperature of bath surrounding enclosure, 12° C.	
Excess of temperature of globe above temperature of surroundings.	Loss of heat per sq. centim. per second per 1° C. of excess.	Excess of temperature of globe above temperature of surroundings.	Loss of heat per sq. centim. per second per 1° C. of excess.
°		°	
240·5	5·58 × 10^{-5}	254·5	8·39 × 10^{-5}*
233·4	5·10	246·6	8·21 *
227·2	4·06	235·1	7·35
222·3	3·07	214·7	7·28
218·7	2·68	199·8	7·18
211·4	2·64	184·3	6·98
208·4	2·55	171·2	6·87
205·7	2·59	159·3	6·66
202·9	2·59	146·0	6·57
198·2	2·16	136·1	6·31
195·9	2·25		
193·6	1·95		
191·5	1·64		
187·7	1·73		
183·9	1·86		
179·9	1·77		
176·6	1·90		
174·9	1·90		
167·9	1·92		
164·5	1·94		
161·2	1·94		

* Vacuum unsatisfactory, improving by working pump. Surface of globe found to be tarnished at conclusion of this experiment.

Appendix.

Added March, 1893.

The method of calculation of the results of my experiments is explained in the body of my paper; but it has been suggested that a more detailed account of the calculations, with, perhaps, a numerical example, might be of value. Accordingly, though I fear it cannot be done without considerable repetition, I beg to supply the want in this appendix.

The numerical results of my experiments are obtained, as has been explained, in the form of readings of the galvanometer deflection taken at equal intervals of time, five minutes being the interval most commonly adopted. The arrangements for making the observations at the precise instant are explained on p. 599 of the paper. These arrangements were perfectly successful. Thanks largely to the skill of Mr. Tanakadate in this kind of observation, it was very rarely that a reading was missed; and when by any chance a reading was missed the failure was known and the exact time of the next observation was noted. There was on no occasion whatever any *uncertainty*.

At suitable times also the temperature of the surrounding vessel, and the state of the vacuum and other particulars, were noted.

While an experiment on the cooling of a globe was in progress the galvanometer readings were plotted on to a large sheet of squared paper. This was done because it was impossible to see into the enclosure; and some check was wanted by which it could be known that everything was proceeding satisfactorily and had been carried out properly in the preparations for the experiment. Usually each experiment had some special feature for which special preparation was made; and it was quite possible that some contact of the wires of the thermo-junction, or some flaw in the last soldering of the halves of the enclosure, should make the result of the long course of waiting and observing valueless. Fortunately no such mischance did occur; but a check was necessary, and the plotting of the curve, which could be quickly compared by eye with others of the same kind, afforded a test which was particularly easy and satisfactory.

The following is a copy of one of the pages of my note-book. It explains itself; as, indeed, each page ought to do :—

April 2, 1890. Observed—J. T. Bottomley, A. Tanakadate. Sooted globe in vacuum; thin coating.

Time.	Deflection.	Temperature of bath.	Vacuum, &c.
h. m.		°	
3 37 P.M.	1171·9	16·0 16·2	Good vacuum.*
40	1034·5	15·7	Vacuum improving.
45	864·6	15·9	Volume 10 Pressure 13 — Readings of volume and pressure on the MacLeod gauge.
50	756·3	..	Volume 10 Pressure 12
55	669·9		
4 00	597·0	15·9	
5	535·9		
10	483·1	..	Volume 10. Pressure 12.
15	436·7	15·9	
20	399·0		
25	365·2		
30	335·4	..	Volume 10. Pressure 12.
35	309·4	15·6 15·9	
40	286·5	15·9	
45	266·5	..	Volume 10. Pressure 12.
50	248·4		
55			
5 00	218·9	15·9	

* Neglect first two readings, vacuum not good enough.

It will be seen that the galvanometer readings are noted opposite particular instants of time. The galvanometer readings, properly corrected, give the temperature of the globe at those instants of time. The differences of the logarithms of these readings give the cooling; and the differences between the time readings give the intervals of time corresponding to the cooling; while also the readings themselves give the differences, at the beginning and end of the time intervals, between the temperatures of the globe and its surroundings, and also some mean between the pairs of readings gives the actual temperature corresponding to the emissivity to be calculated. The object of the subsequent calculation was to find the absolute

emissivity at the temperatures and under the circumstances noted; but neither the absolute time nor the intervals of time appear in the final result.

In order to prepare the numbers thus recorded for calculation it was necessary to apply certain small corrections to the galvanometer readings.

1. As the reflecting galvanometer and a plane scale were used, and as the angle through which the ray of light turns is double that through which the mirror and magnet are turned by the current force, a number proportional to n^3/d^2 must be subtracted from the galvanometer readings in order to obtain numbers proportional to the magnet deflections (or their sines or tangents); n being the observed deflection and d the distance of the scale from the mirror. To do this a table of values of $n^3/(10152)^2$ was prepared; but very convenient tables for this purpose were published in 1890 by Dr. PAUL CZERMAK,* and ought now to form a part of the equipment of a physical laboratory.

2. The second correction consists in the subtraction of a small quantity proportional to the square of the deflection. It is for the term in the thermo-junction formula which is proportional to the square of the temperature. This correction was also applied with the help of a table or with the slide rule. (See footnote to p. 603 of the paper.)

These subtractions having been made, I obtain what I may call the "proportional deflections" and these stand opposite to the times observed. These numbers are proportional to the differences of the temperatures of the globe and the water-bath surrounding its enclosure at the corresponding times. The following copy from pages of my calculation book will show all details.

* Footnote to p. 597 of paper.

CALCULATION of Results of April 2, 1890.

Sooted globe in vacuum ; thin coating. Vacuum $\frac{1}{1\cdot27}$ M. (Apparatus, fig. 2.)

(1) Time.	(2) Observed deflection.	(3) Corrections and sum of corrections.	(4) Proportional deflection.	(5) Log. of proportional deflection.	(6) Differences of logs. of proportional deflections.	(7) Emissivity (calculated by multiplying numbers of column 6 by $4\cdot323\times10^{-3}$)	(8) Log. of difference of temperatures of globe and surroundings.	(9) Difference of temperatures of globe and surroundings.
P.M. 3.40	1034·5	10·7 168·0 178·7	855·8	2·9324	·0643	$2\cdot780\times10^{-4}$	2·3792	239°·4
3.45	864·6	6·6 120·0 126·6	738·0	·8681	·0468	2·024	2·3149*	206·5
3.50	756·3	4·2 89·5 93·7	662·6	·8213	·0453	1·959	2·2681	185·4
3.55	669·9	2·9 70·0 72·9	597·0	·7760	·0438	1·894	2·2228	167·0
4.0	597·0	2·0 55·2 57·2	539·8	·7322	·0422	1·824	2·1790	151·0
4.5	535·9	1·5 44·6 46·1	489·8	·6900	·0408	1·764	2·1368	137·0
4.10	483·1	1·1 36·2 37·3	445·8	·6492	·0397	1·716	2·0960	124·7
4.15	436·7	0·8 29·0 29·8	406·9	·6095	·0372	1·608	2·0563	113·9
4.20	399·0	0·6 24·9 25·5	373·5	·5723	·0362	1·565	2·0191	104·5
4.25	365·2	0·5 21·1 21·6	343·6	·5361	·0347	1·500	1·9829	96·14
4.30	335·4	0·4 17·8 18·2	317·2	·5014	·0331	1·431	1·9482	88·76
4.35	309·4	0·3 15·1 15·4	294·0	·4683	·0317	1·370	1·9151	82·24
4.40	286·5	0·2 13·1 13·3	273·2	·4366	·0301	1·301	1·8834	76·45
4.45	266·5	0·2 11·3 11·5	255·0	·4065	·0286	1·237	1·8533	71·34
4.50	248·4	0·1 9·6 9·7	238·7	·3779			1·8247	66·79
4.55					·0526	1·137		
5.0	218·9	0·1 7·3 7·4	211·5	·3253			1·7721	59·17

* Vacuum not good enough at first reading or first and second.

Taking the common logarithms of the numbers in column 4 we have the fifth column of the table on p. 615.

The differences between the successive logarithms of deflections gives col. 6. To obtain the proper emissivities, these numbers must be multiplied by $(M \times c)/(300 \times S)$; where M is the modulus for reducing Neperian logarithms to common logarithms, c the capacity for heat of the globe, S the cooling surface, and 300 the interval of time in seconds. (See p. 604 of the paper.) The value of $Mc/300\,S$ is $4{\cdot}323 \times 10^{-3}$. The multiplication I perform by means of CRELLE'S table.

Lastly, to find the difference of temperatures of globe and bath corresponding to the emissivities, I use the logarithms of the deflections in col. 5, and to each of these add the logarithm of the multiplier corresponding to the first term of the thermo-junction formula. In the formula of the note of p. 603 of the paper, this multiplier would be $\frac{1}{3{\cdot}38}$ and the *additive* logarithm $0{\cdot}4701 - 1$. In the present case April 2, 1890, the multiplier is $0{\cdot}27976$; and the additive logarithm $0{\cdot}4468 - 1$. Taking the anti-logarithms of the numbers so found, I obtain the difference of temperatures in Centigrade degrees between cooling body and surroundings.

Fig. 1.

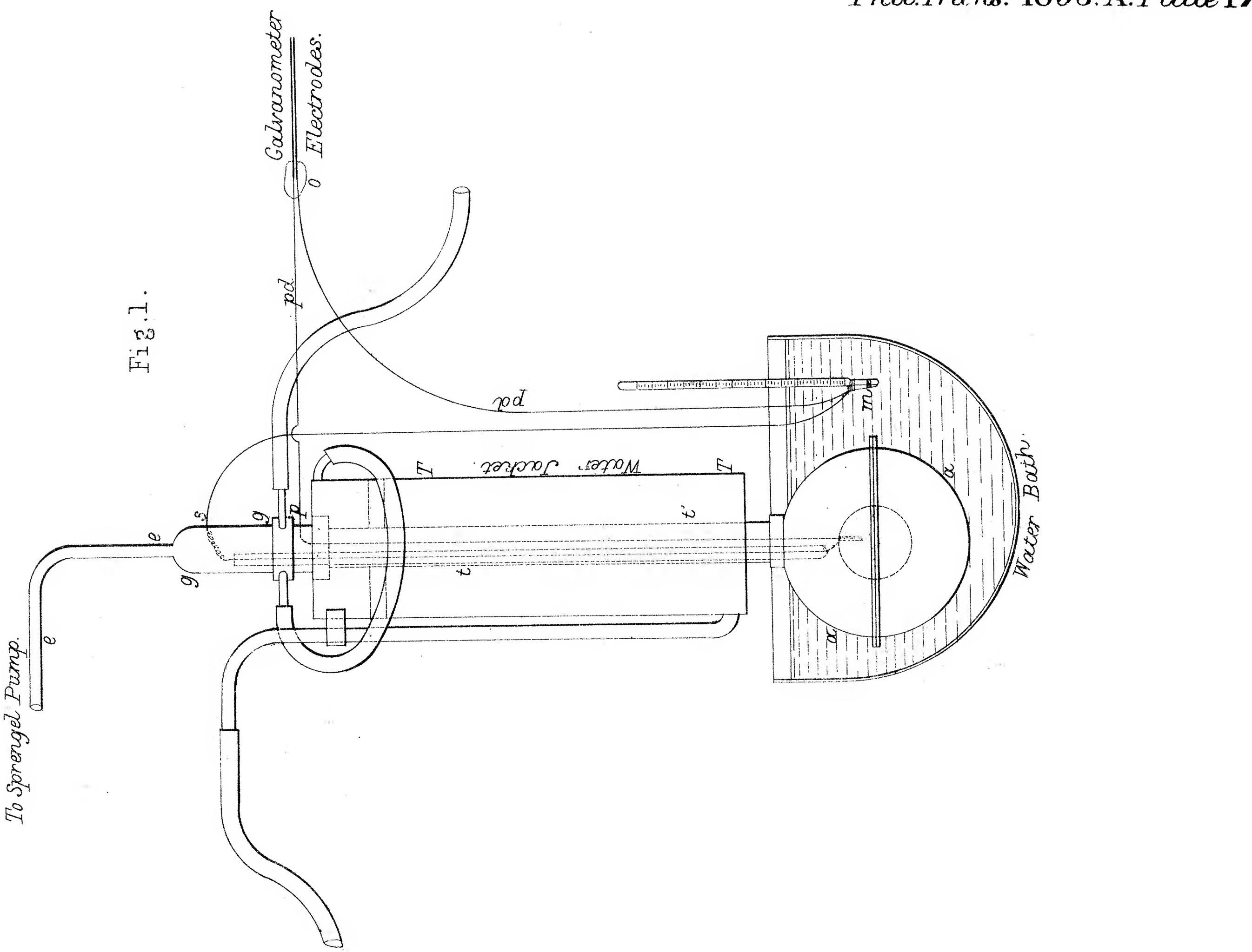
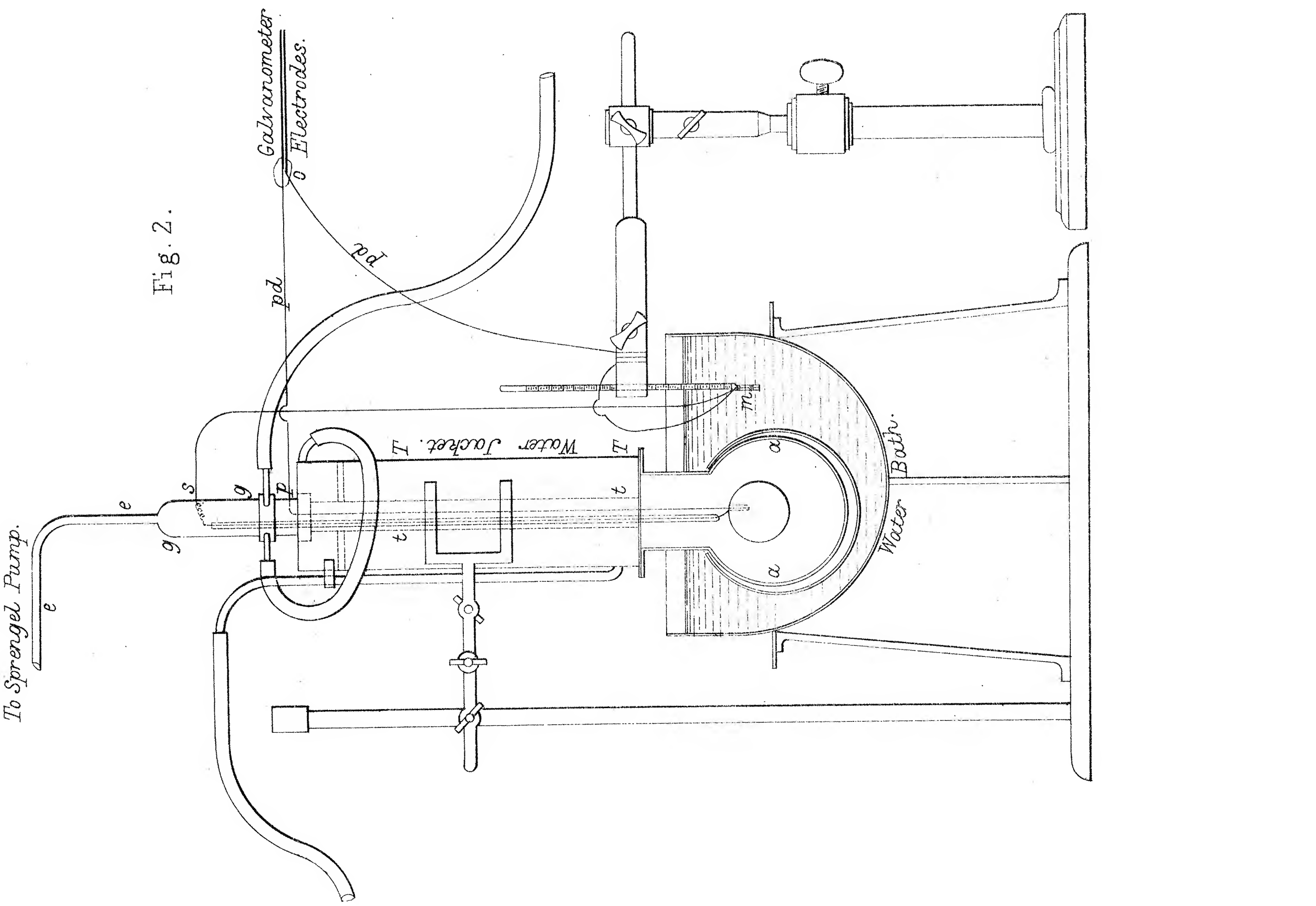

To Sprengel Pump.
Galvanometer
Electrodes.
pd
pd
Water Jacket.
Water Bath.
T
T
t
t'
g
g
s
e
e
p
m
o
α
α

Fig. 2.


To Sprengel Pump.
Galvanometer
Electrodes.
pd
pd
Water Jacket.
Water Bath.
T
T
t
t
g
g
s
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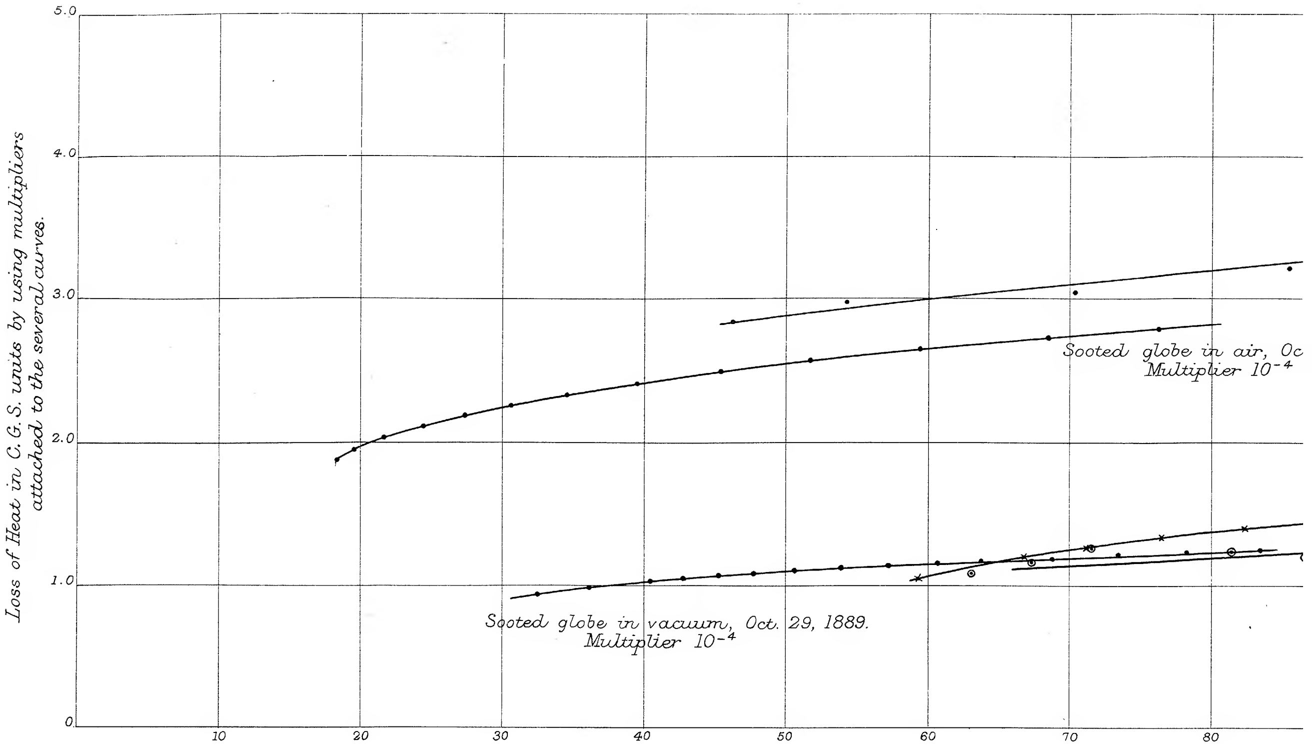
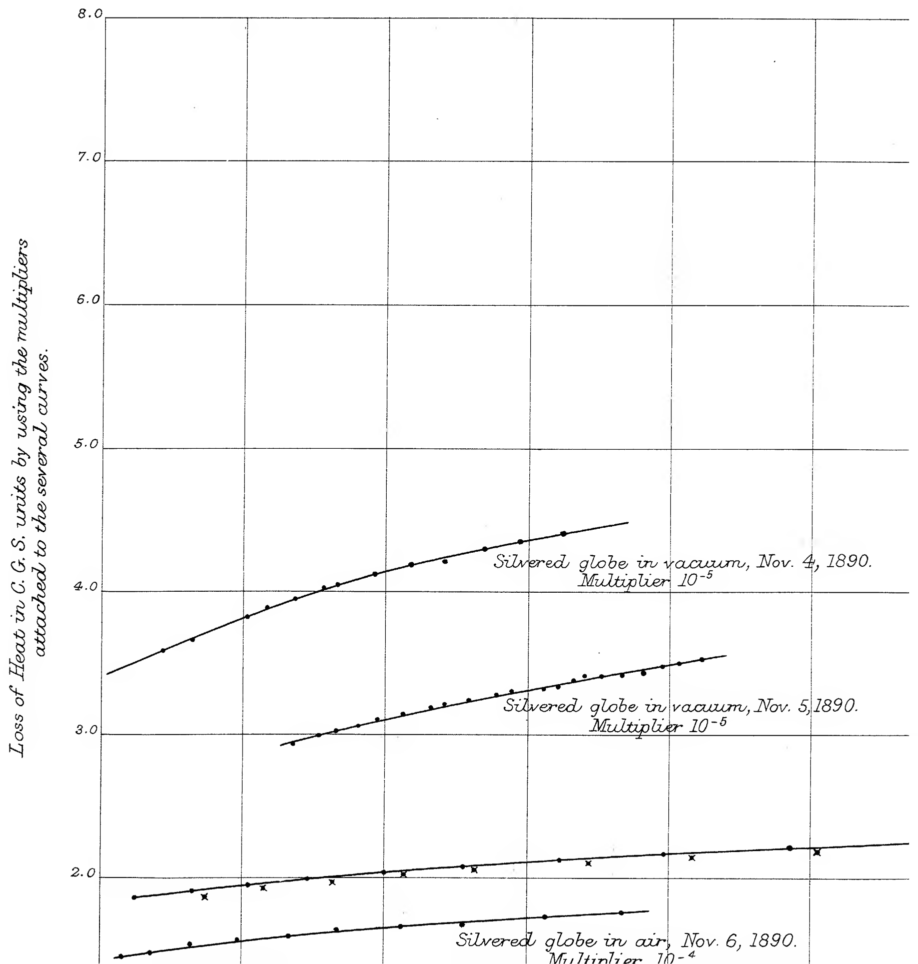
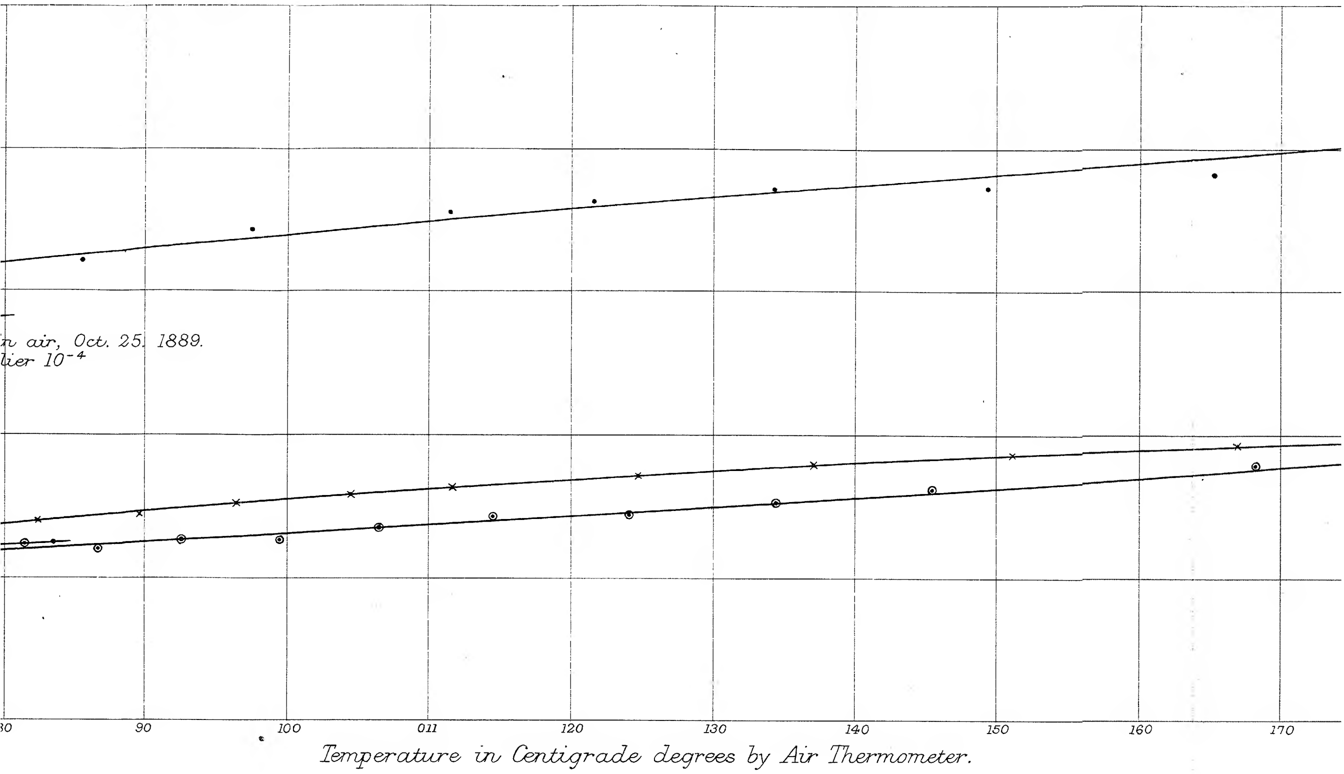
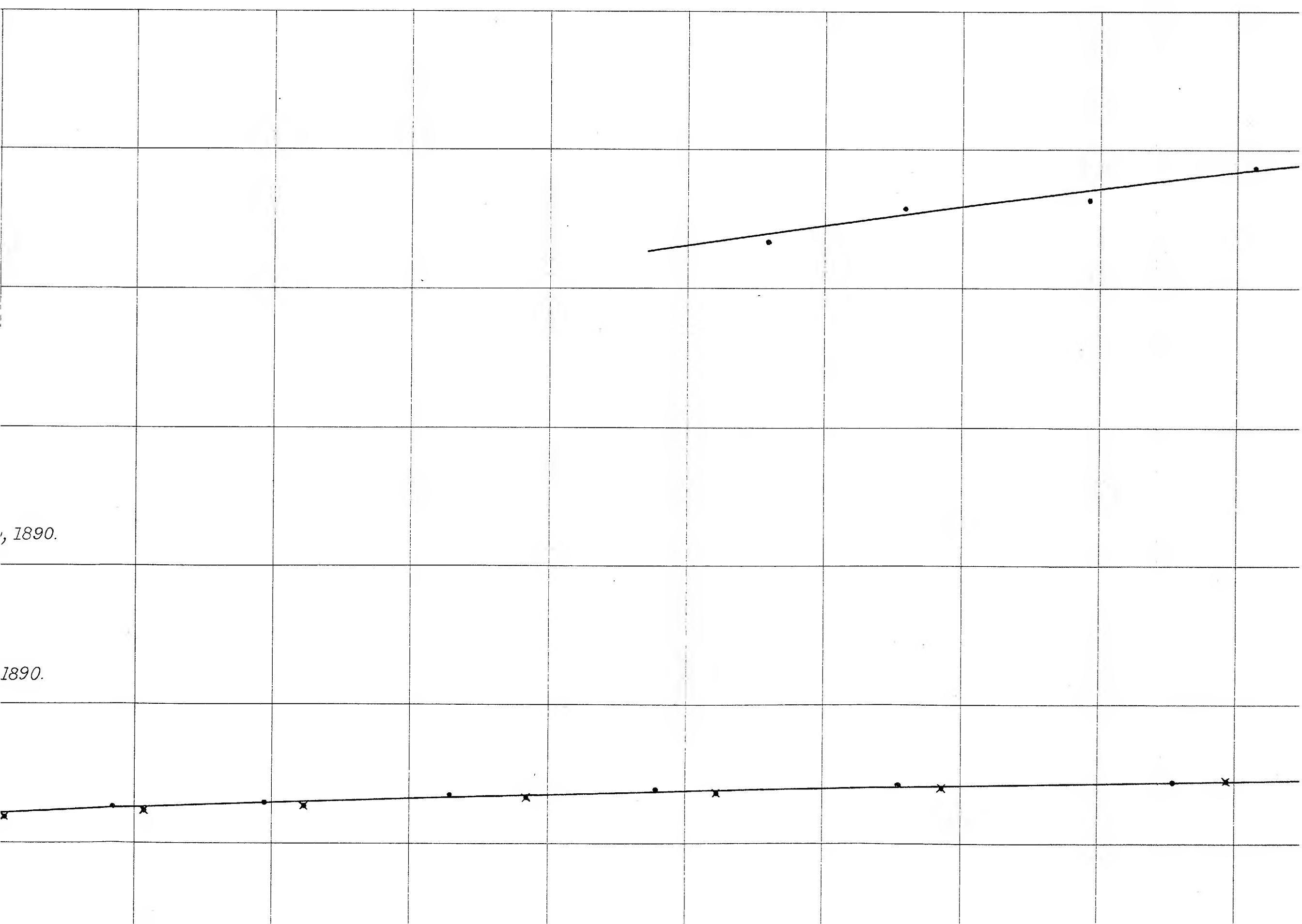
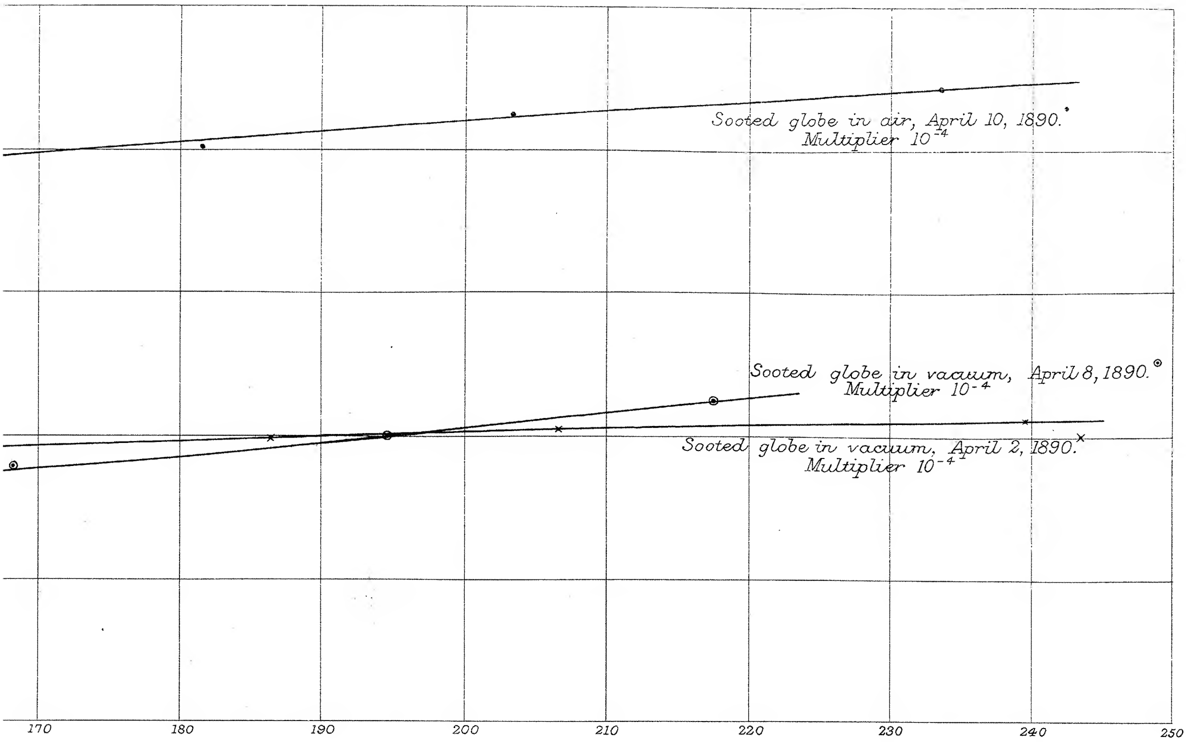
e
p
m
o
α
α

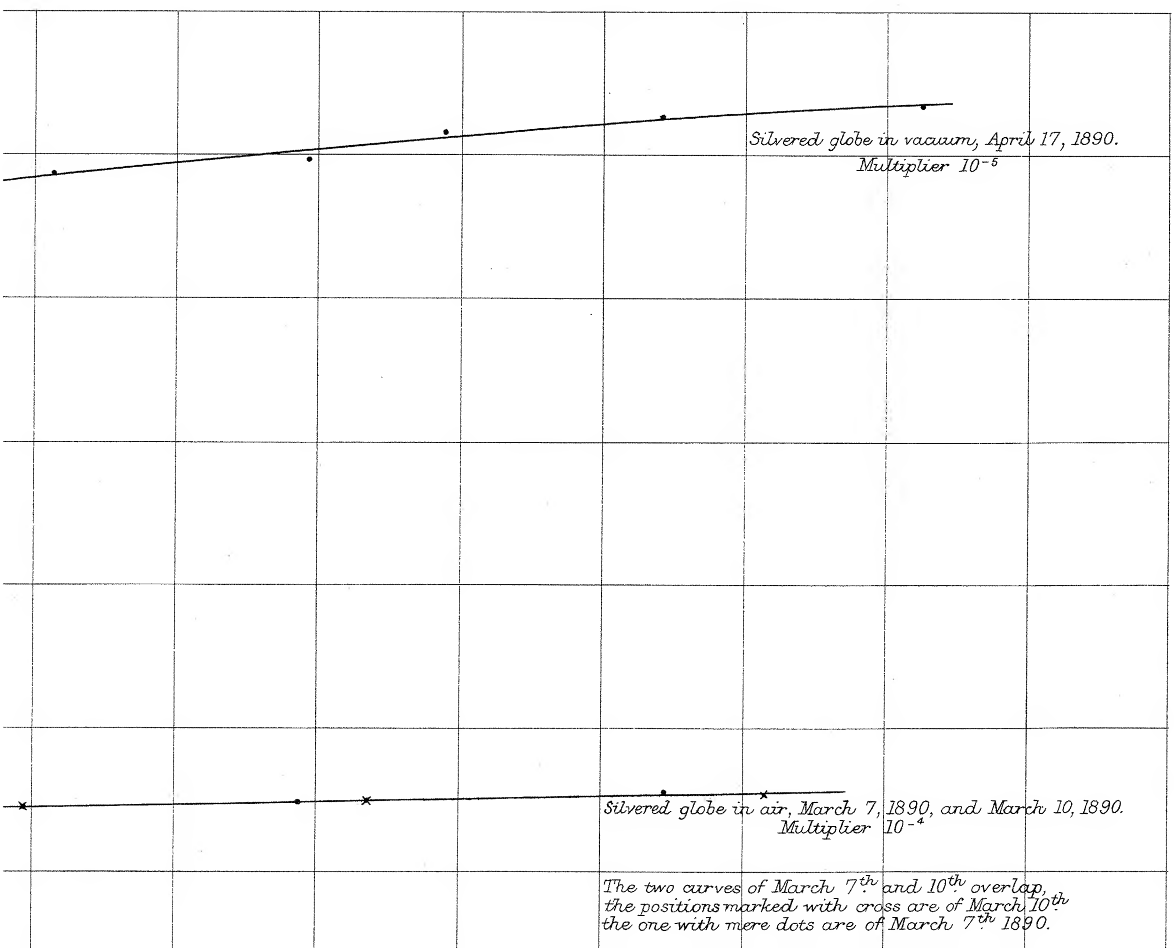

Loss of Heat in C. G. S. units by using multipliers attached to the several curves.
Sooted globe in air, Oc
Multiplier 10^{-4}
Sooted globe in vacuum, Oct. 29, 1889.
Multiplier 10^{-4}
5.0
4.0
3.0
2.0
1.0
0
10
20
30
40
50
60
70
80

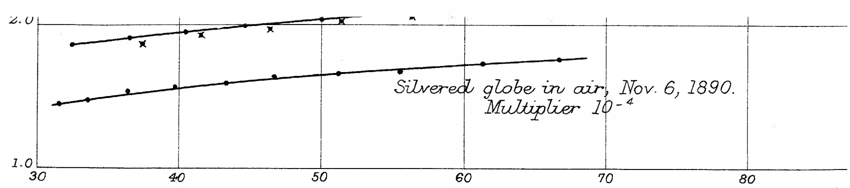

Loss of Heat in C. G. S. units by using the multipliers attached to the several curves.
Silvered globe in vacuum, Nov. 4, 1890.
Multiplier 10^{-5}
Silvered globe in vacuum, Nov. 5, 1890.
Multiplier 10^{-5}
Silvered globe in air, Nov. 6, 1890.
Multiplier 10^{-4}
8.0
7.0
6.0
5.0
4.0
3.0
2.0


n air, Oct. 25. 1889.
lier 10-4
90
100
011
120
130
140
150
160
170
Temperature in Centigrade degrees by Air Thermometer.

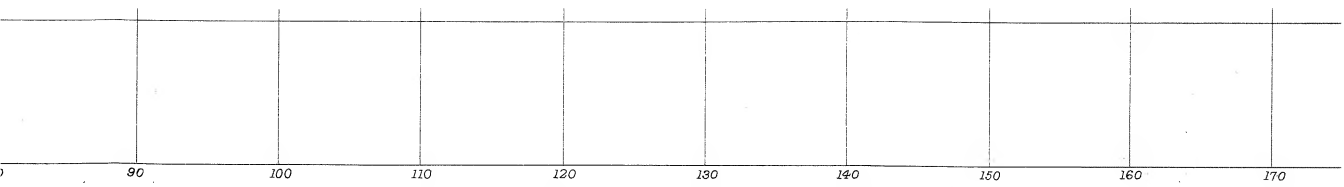

, 1890.
1890.


Sooted globe in air, April 10, 1890.
Multiplier 10^{-4}
Sooted globe in vacuum, April 8, 1890.
Multiplier 10^{-4}
Sooted globe in vacuum, April 2, 1890.
Multiplier 10^{-4}
170
180
190
200
210
220
230
240
250

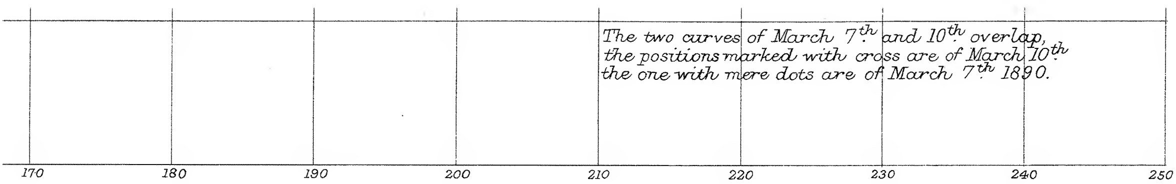

Silvered globe in vacuum, April 17, 1890.
Multiplier 10^{-5}
Silvered globe in air, March 7, 1890, and March 10, 1890.
Multiplier 10^{-4}
The two curves of March 7th and 10th overlap,
the positions marked with cross are of March 10th
the one with mere dots are of March 7th 1890.



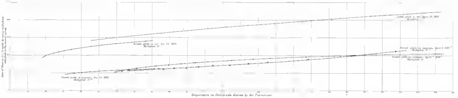
2.0
1.0
30
40
50
60
70
80
Silvered globe in air, Nov. 6, 1890.
Multiplier 10^{-4}

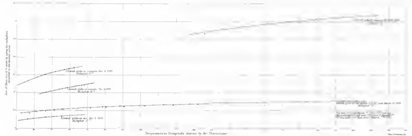


Temperature in Centigrade degrees by Air Thermometer.



West, Newman, lith.


Temperature in Centigrade degrees by Air Thermometer


Temperature in Centigrade degrees by Air Thermometer